Distributional Semantics and Word Embeddings

Chase Geigle

2016-10-14

What is a word?

“dog”

What is a word?

“dog” “canine”

What is a word?

“dog”

“canine”

3

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“dog”	“canine”
3	399,999

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$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

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3	399,999
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What is a word?

“dog”	“canine”
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$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ 0 \end{pmatrix}$$

Sparsity!

The Scourge of Sparsity

In the real world, **sparsity** hurts us:

- Low vocabulary coverage in training $\rightarrow$ high OOV rate in applications (poor generalization)
- “one-hot” representations $\rightarrow$ *huge* parameter counts
- information about one word *completely unutilized* for another!

Thus, a good representation must:

- reduce parameter space,
- improve generalization, and
- somehow “transfer” or “share” knowledge between words

The Distributional Hypothesis

You shall know a word by the company it keeps.
(J. R. Firth, 1957)

Words with **high similarity** occur in the **same contexts** as one another.

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A bit of foreshadowing:

- A **word** ought to be able to **predict its context** (word2vec Skip-Gram)
- A **context** ought to be able to **predict its missing word** (word2vec CBOW)

Brown Clustering

Language Models

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- n-gram language model: $p(w_n \mid w_1^{n-1}, \theta)$

$$p(\mathbf{W} \mid \theta) = \prod_{i=1}^{N} p(w_i \mid w_{i-n+1}^{i-1}, \theta)$$

A Class-Based Language Model[1]

Main Idea: cluster words into a fixed number of clusters C and use their cluster assignments as their identity instead (reducing sparsity)

If $\pi : V \to C$ is a mapping function from a word type to a cluster (or class), we want to find

$$\pi^* = \arg\max p(\mathbf{W} \mid \pi)$$

where

$$p(\mathbf{W} \mid \pi) = \prod_{i=1}^{N} p(c_i \mid c_{i-1}) p(w_i \mid c_i).$$

[1] Peter F. Brown et al. "Class-based N-gram Models of Natural Language". In: *Comput. Linguist.* 18.4 (Dec. 1992), pp. 467–479.

Finding the best partition

$$\pi^* = \arg\max_{\pi} P(\mathbf{W} \mid \pi) = \arg\max_{\pi} \log P(\mathbf{W} \mid \pi)$$

One can derive[2] $L(\pi)$ to be

$$L(\pi) = \underbrace{\sum_{w} n_w \log n_w}_{\substack{\text{(nearly) unigram entropy} \\ \text{(fixed w.r.t. } \pi)}} + \underbrace{\sum_{c_i, c_j} n_{c_i, c_j} \log \frac{n_{c_i, c_j}}{n_{c_i} \cdot n_{c_j}}}_{\substack{\text{(nearly) mutual information} \\ \text{(varies with } \pi)}}$$

[2] Sven Martin, Jörg Liermann, and Hermann Ney. "Algorithms for Bigram and Trigram Word Clustering". In: *Speech Commun.* 24.1 (Apr. 1998), pp. 19–37.

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What does maximizing this mean?

Recall

$$MI(c, c') = \sum_{c_i, c_j} p(c_i, c_j) \log \frac{p(c_i, c_j)}{p(c_i) p(c_j)}$$

which can be shown to be

$$MI(c, c') = \underbrace{\frac{1}{N}}_{\text{(constant)}} \sum_{c_i, c_j} n_{c_i, c_j} \log \frac{n_{c_i, c_j}}{n_{c_i} \cdot n_{c_j}} + \underbrace{\log N}_{\text{(constant)}}$$

and thus **maximizing MI of adjacent classes** selects the best π.

Finding the best partition (cont'd)

$$\pi^* = \arg\max_{\pi} \underbrace{\sum_{w} n_w \log n_w}_{\substack{\text{(nearly) unigram entropy} \\ \text{(fixed w.r.t. } \pi\text{)}}} + \underbrace{\sum_{c_i, c_j} n_{c_i, c_j} \log \frac{n_{c_i, c_j}}{n_{c_i} \cdot n_{c_j}}}_{\substack{\text{(nearly) mutual information} \\ \text{(varies with } \pi\text{)}}}$$

Direct maximization is **intractable!** Thus, agglomerative (bottom-up) clustering is used as a greedy heuristic.

The best merge is determined by the lowest loss in average mutual information.

Agglomerative Clustering

Do they work? (Yes.)

Named entity recognition:

- Scott Miller, Jethran Guinness, and Alex Zamanian. "Name Tagging with Word Clusters and Discriminative Training." In: *HLT-NAACL*. 2004, pp. 337–342
- Lev Ratinov and Dan Roth. "Design Challenges and Misconceptions in Named Entity Recognition". In: *Proc. CoNLL*. 2009, pp. 147–155

Dependency parsing:

- Terry Koo, Xavier Carreras, and Michael Collins. "Simple Semi-supervised Dependency Parsing". In: *Proc. ACL: HLT*. June 2008, pp. 595–603
- Jun Suzuki and Hideki Isozaki. "Semi-Supervised Sequential Labeling and Segmentation Using Giga-Word Scale Unlabeled Data". In: *Proc. ACL: HLT*. June 2008, pp. 665–673

Constituency parsing:

- Marie Candito and Benoît Crabbé. "Improving Generative Statistical Parsing with Semi-supervised Word Clustering". In: IWPT. 2009, pp. 138–141
- Muhua Zhu et al. "Fast and Accurate Shift-Reduce Constituent Parsing". In: *ACL*. Aug. 2013, pp. 434–443

Vector Spaces for Word Representation

Toward Vector Spaces

Brown clusters are nice, but limiting:

- Arbitrary cut-off point $\rightarrow$ two words may be assigned different classes arbitrarily because of our chosen cutoff
- Definition of "similarity" is limited to only bigrams of classes[3]
- Completely misses lots of **regularity** that's present in language

[3] There *are* adaptations of Brown clustering that move past this, but what we'll see today is still even better.

What do I mean by “regularity”?

woman is to **sister** as **man** is to

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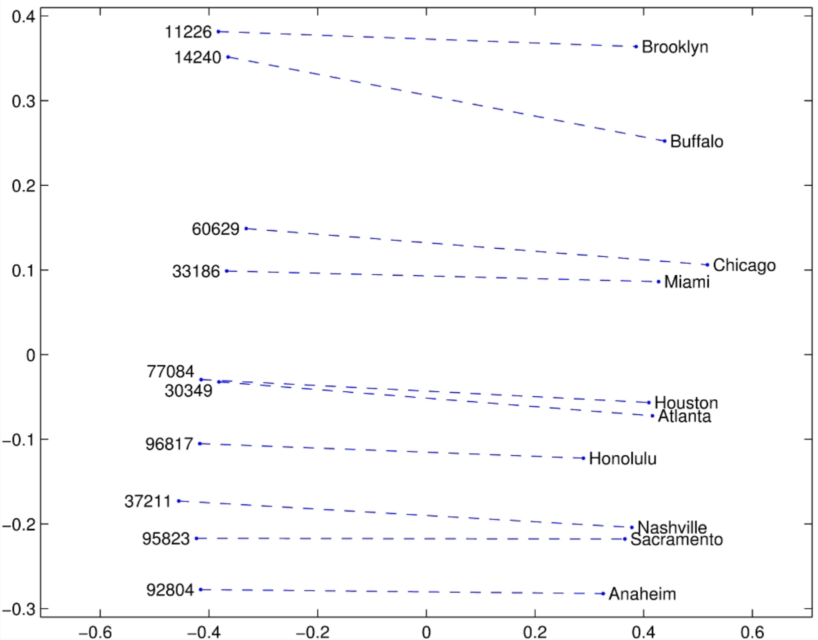
The **differences** between each **pair of words** are similar.

Can our word representations capture this? (demo)



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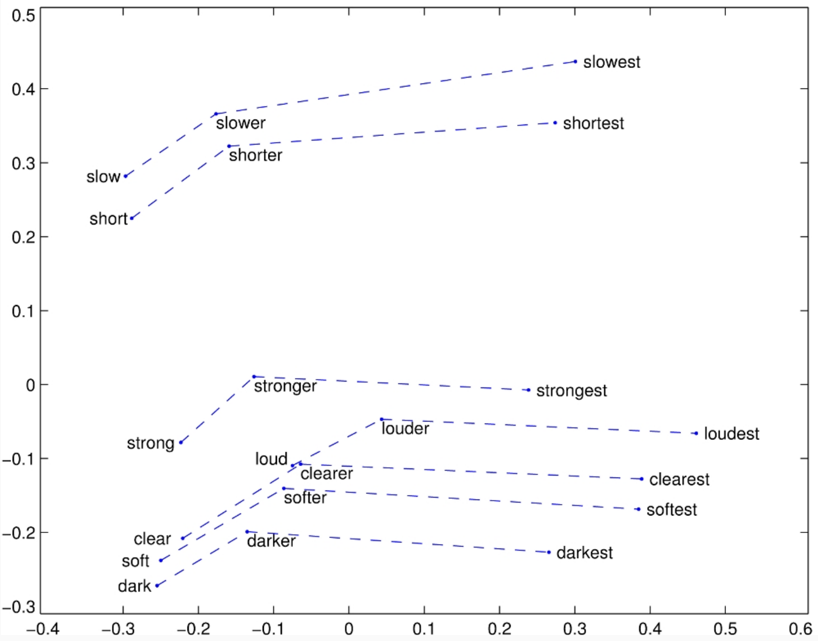
slowest
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Neural Word Embeddings

Associate a low-dimensional, dense vector $\vec{w}$ with each word $w \in V$ so that similar words (in a distributional sense) share a similar vector representation.

If I only had a vector: The Analogy Problem

To solve analogy problems of the form "w_a is to w_b as w_c is to what?", we can simply compute a query vector

$$\mathbf{q} = \mathbf{w}_b - \mathbf{w}_a + \mathbf{w}_c$$

and find the most similar word vector $\mathbf{v} \in \mathbf{W}$ to $\mathbf{q}$. If we normalize $\mathbf{q}$ to unit-length

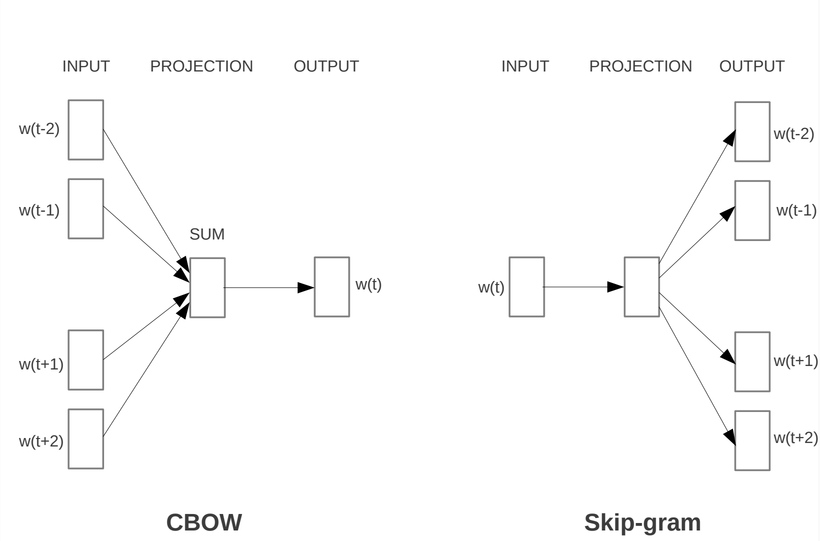
$$\hat{\mathbf{q}} = \frac{\mathbf{q}}{||\mathbf{q}||}$$

and assume each vector in $\mathbf{W}$ is also unit-length, this reduces to computing

$$\arg\max_{\mathbf{v} \in \mathbf{W}} \mathbf{v} \cdot \hat{\mathbf{q}}$$

and returning the associated word v.

The CBOW and Skip-Gram Models[4] (word2vec)



[4] Tomas Mikolov et al. "Efficient estimation of word representations in vector space". In: *ICLR Workshop* (2013).

Skip-Gram, Mathematically

Training corpus $w_1, w_2, \ldots, w_N$ (with N typically in the billions) from a fixed vocabulary V.

Goal is to maximize the average log probability:

$$\frac{1}{N} \sum_{i=1}^{N} \sum_{-L \leq k \leq L; k \neq 0} \log p(w_{i+k} \mid w_i)$$

Associate with each $w \in V$ an "input vector" $\mathbf{w} \in \mathbb{R}^d$ and an "output vector" $\tilde{\mathbf{w}} \in \mathbb{R}^d$. Model context probabilities as

$$p(c \mid w) = \frac{\exp(\mathbf{w} \cdot \tilde{\mathbf{c}})}{\sum_{c' \in V} \exp(\mathbf{w} \cdot \tilde{\mathbf{c}}')}.$$

The problem? V is *huge*! $\nabla \log p(c \mid w)$ takes time $O(|V|)$ to compute!

Negative Sampling[5]

Given a pair (w, c), can we determine if this came from our corpus or not? Model probabilistically as

$$p(D = 1 \mid w, c) = \sigma(\mathbf{w} \cdot \tilde{\mathbf{c}}) = \frac{1}{1 + \exp(-\mathbf{w} \cdot \tilde{\mathbf{c}})}.$$

Goal: maximize $p(D = 1 \mid w, c)$ for pairs (w, c) that **occur in the data**.

Also maximize $p(D = 0 \mid w, c_N)$ for (w, c_N) pairs where c_N is **sampled randomly** from the empirical unigram distribution.

[5] Tomas Mikolov et al. "Distributed Representations of Words and Phrases and their Compositionality". In: *Advances in Neural Information Processing Systems 26*. 2013, pp. 3111–3119.

Skip-Gram Negative Sampling Objective

Locally,

$$\ell = \log \sigma(\mathbf{w} \cdot \tilde{\mathbf{c}}) + k \cdot \mathbb{E}_{c_N \sim p_D(c_N)} \left[\log \sigma(-\mathbf{w} \cdot \tilde{\mathbf{c_N}})\right]$$

and thus globally

$$\mathcal{L} = \sum_{w \in V} \sum_{c \in V} (n_{w,c}) \left(\log \sigma(\mathbf{w} \cdot \tilde{\mathbf{c}}) + k \cdot \mathbb{E}_{c_N \sim P_n(c_N)} \left[\log \sigma(-\mathbf{w} \cdot \tilde{\mathbf{c_N}})\right]\right)$$

- k: number of negative samples to take (hyperparameter)
- $n_{w,c}$: number of times (w, c) was seen in the data
- $\mathbb{E}_{c_N \sim P_n(c_N)}$ indicates an expectation taken with respect to the noise distribution $P_n(c_N)$.

Implementation Details (Read: Useful Heuristics)

1. What is the noise distribution?

$$P_n(c_N) = \frac{(n_{c_N}/N)^{3/4}}{Z}$$

   (which is the empirical unigram distribution raised to the 3/4 power).

2. Frequent words can dominate the loss. Throw away word w_i in the training data according to

$$P(w_i) = 1 - \sqrt{\frac{t}{n_{w_i}}}$$

   where t is some threshold (like 10^{-5}).

Both are essentially unexplained by Mikolov et al.

What does this *actually do*?

It turns out this is nothing that new! Levy and Goldberg[6] show that SGNS is **implicitly factorizing** the matrix

$$M_{i,j} = \mathbf{w_i} \cdot \tilde{\mathbf{c}}_\mathbf{j} = PMI(w_i, c_j) - \log k = \log \frac{p(w_i, c_j)}{p(w_i)p(c_j)} - \log k$$

using an objective that weighs deviations in more frequent (w, c) pairs more strongly than less frequent ones.

Thus...

[6] Omer Levy and Yoav Goldberg. “Neural Word Embedding as Implicit Matrix Factorization”. In: *Advances in Neural Information Processing Systems 27*. 2014, pp. 2177–2185.

Matrix Factorization Methods for Word Embeddings

Why not just SVD?

Since SGNS is just factorizing a shifted version of the PMI matrix, **why not just perform a rank-k SVD** on the PMI matrix directly?

Assignment 6!

With a little TLC, this method can actually work and does surprisingly well.

GloVe: Global Vectors for Word Representation[7]

How can we capture words related to **ice**, but not to **steam**?

Prob. or Ratio	w_k = **solid**	w_k = **gas**	w_k = **water**	w_k = **fashion**
$P(w_k \mid \text{ice})$	1.9×10^{-4}	6.6×10^{-5}	3.0×10^{-3}	1.7×10^{-5}
$P(w_k \mid \text{steam})$	2.2×10^{-5}	7.8×10^{-4}	2.2×10^{-3}	1.8×10^{-5}
$\frac{P(w_k \mid \text{ice})}{P(w_k \mid \text{steam})}$	8.9	8.5×10^{-2}	1.36	0.96

Probability ratios are most informative:

- **solid** is related to **ice** but not **steam**
- **gas** is related to **steam** but not **ice**
- **water** and **fashion** do not discriminate between **ice** or **steam** (ratios close to 1)

[7] Jeffrey Pennington, Richard Socher, and Christopher D. Manning. "GloVe: Global Vectors for Word Representation". In: *EMNLP*. 2014, pp. 1532–1543.

Building a model: intuition

We would like for the vectors $\mathbf{w}_i$, $\mathbf{w}_j$, and $\mathbf{w}_k$ to be able to capture the information present in the probability ratio. If we set

$$\mathbf{w}_i^\intercal \mathbf{w}_k = \log P(w_k \mid w_i)$$

then we can easily see that

$$(\mathbf{w}_i - \mathbf{w}_j)^\intercal \mathbf{w}_k = \log P(w_k \mid w_i) - \log P(w_k \mid w_j) = \log \frac{P(w_k \mid w_i)}{P(w_k \mid w_j)},$$

ensuring that the information present in the co-occurrence probability ratio is expressed in the vector space.

GloVe Objective

$$\mathbf{w}_i^\intercal \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j = \log X_{ij}$$

Again: two sets of vectors: "target" vectors $\mathbf{w}$ and "context" vectors $\tilde{\mathbf{w}}$.

$X_{ij} = n_{w_i, w_j}$ is the number of times w_j appears in the context of w_i.

Problem: all co-occurrences are weighted equally

$$J = \sum_{i,j=1}^{V} f(X_{ij})(\mathbf{w}_i^\intercal \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$

Final objective is just **weighted least-squares**. $f(X_{ij})$ serves as a "dampener", lessening the weight of the rare co-occurrences.

$$f(x) = \begin{cases} \left(\frac{x}{x_{max}}\right)^\alpha & \text{if } x < x_{max} \\ 1 & \text{otherwise} \end{cases}$$

where α's default is (you guessed it) 3/4, and x_{max}'s default is 100.

Relation to word2vec

GloVe Objective:

$$J = \sum_{i,j=1}^{V} f(X_{ij})(\mathbf{w}_i^\intercal \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$

word2vec (Skip-Gram) Objective (after rewriting):

$$J = -\sum_{i=1}^{V} X_i \sum_{j=1}^{V} P(w_j \mid w_i) \log Q(w_j \mid w_i)$$

where $X_i = \sum_k X_{ik}$ and P and Q are the empirical co-occurrence and model co-occurrence distributions, respectively. **Weighted cross-entropy error!**

Authors show that **replacing cross-entropy with least-squares nearly re-derives GloVe**:

$$\hat{J} = \sum_{i,j} X_i (w_i^\intercal \tilde{w}_j - \log X_{ij})^2$$

Model Complexity

word2vec: $O(|C|)$, linear in corpus size

GloVe naïve estimate: $O(|V|^2)$, square of vocab size

But it actually only depends on the number of non-zero entries in $\mathbf{X}$. If co-occurrences are modeled via a power law, then we have

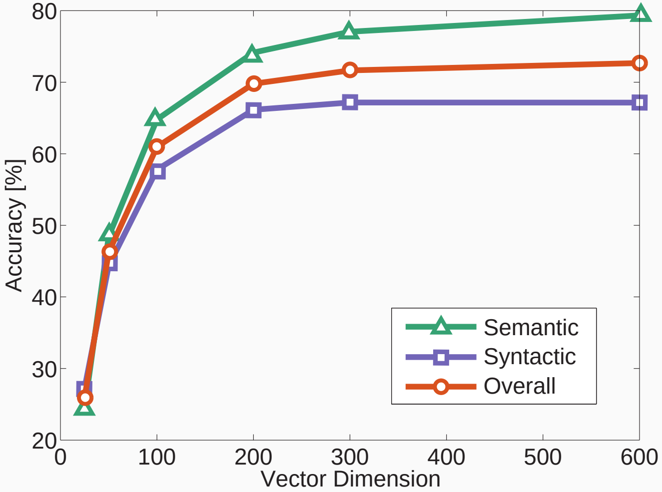
$$X_{ij} = \frac{k}{(r_{ij})^\alpha}.$$

Modeling $|C|$ under this assumption, the authors eventually arrive at

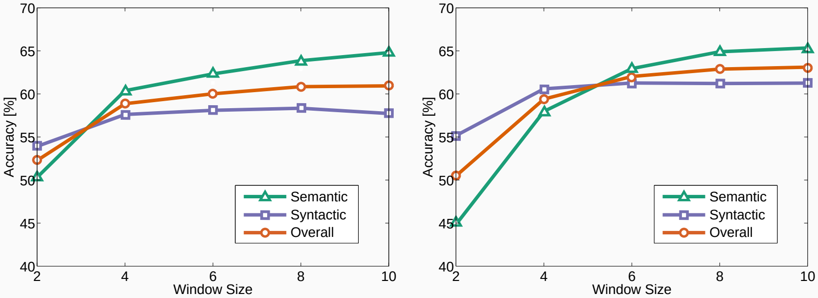
$$|\mathbf{X}| = \begin{cases} O(|C|) & \text{if } \alpha < 1 \\ O(|C|^{1/\alpha}) & \text{otherwise} \end{cases}$$

where the corpora studied in the paper were well modeled with $\alpha = 1.25$, leading to $O(|C|^{0.8})$. In practice, it's faster than word2vec.

Accuracy vs Vector Size

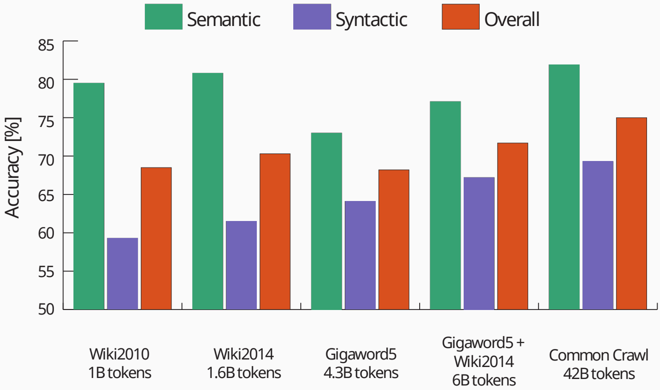


Accuracy vs Window Size

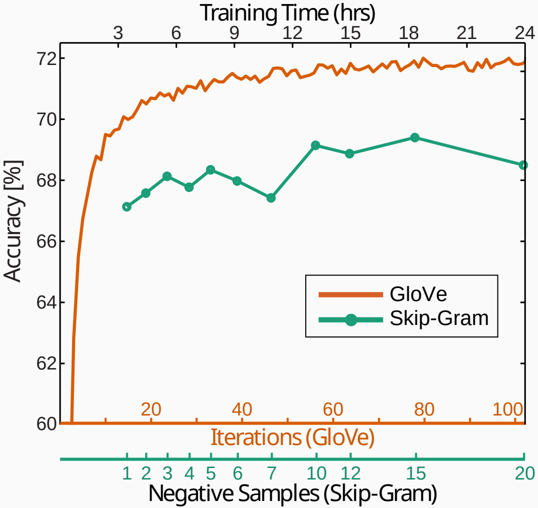


Left: Symmetric window, **Right:** Asymmetric window

Accuracy vs Corpus Choice



Training Time



This evaluation is **not quite fair**!

But wait…

(Referring to `word2vec`)

> *…the code is designed only for a single training epoch…*
> *…specifies a learning schedule specific to one pass through the data, making a modification for multiple passes a non-trivial task.*

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(Referring to `word2vec`)

> *...the code is designed only for a single training epoch...*
> *...specifies a learning schedule specific to one pass through the data, making a modification for multiple passes a non-trivial task.*

This is **false**, and a **bad excuse**.

How could you modify `word2vec` to support multiple “epochs” with zero effort?

But wait...

...we choose to use the sum $\mathbf{W} + \tilde{\mathbf{W}}$ *as our word vectors.*

But wait...

> *...we choose to use the sum* $\mathbf{W} + \tilde{\mathbf{W}}$ *as our word vectors.*

`word2vec` *also* learns $\tilde{\mathbf{W}}$! Why not do the same for both?

Embedding performance is very much a function of your hyperparameters!

Investigating Hyperparameters[8]

Levy, Goldberg, and Dagan perform a **systematic evaluation** of `word2vec`, SVD, and GloVe where **hyperparameters are controlled for** and **optimized across different methods** for a variety of tasks.

Interesting results:

> *MSR's analogy dataset is the only case where SGNS and GloVe substantially outperform PPMI and SVD.*

And:

> *...SGNS outperforms GloVe in **every** task.*

[8] Omer Levy, Yoav Goldberg, and Ido Dagan. "Improving Distributional Similarity with Lessons Learned from Word Embeddings". In: *Transactions of the Association for Computational Linguistics* 3 (2015), pp. 211–225.

But wait... (mark II)

Both GloVe and `word2vec` use **context window weighting**:

- `word2vec`: **samples a window size** $\ell \in [1, L]$ for each token before extracting counts
- GloVe: weighs contexts by their **distance from the target word** using the harmonic function $\frac{1}{d}$

What is the impact of using the different context window strategies between methods?

Levy, Goldberg, and Dagan **did not** investigate this, instead using `word2vec`'s method across all of their evaluations.

Evaluating things properly is non-obvious and often very difficult!

What You Should Know

What You Should Know

- What is the **sparsity problem** in NLP?
- What is the **distributional hypothesis**?
- How does **Brown clustering** work? What is it trying to maximize?
- What is **agglomerative clustering**?
- What is a **word embedding**?
- What is the difference between the **CBOW** and **Skip-Gram** models?
- What is **negative sampling** and why is it useful?
- What is the **learning objective** for **SGNS**?
- What is the matrix that **SGNS is implicitly factorizing**?
- What is the **learning objective** for **GloVe**?
- What are some examples of **hyperparameters for word embedding methods**?

Implementations

- `word2vec`:
  - The **original implementation**: `https://code.google.com/archive/p/word2vec/`
  - Yoav Goldberg's `word2vecf` **modification (multiple epochs, arbitrary context features)**: `https://bitbucket.org/yoavgo/word2vecf`
  - **Python implementation** in `gensim`: `https://radimrehurek.com/gensim/models/word2vec.html`
- GloVe:
  - The **original implementation**: `http://nlp.stanford.edu/projects/glove/`
  - MeTA: `https://meta-toolkit.org`
  - **R implementation** `text2vec`: `http://text2vec.org/`

Thanks!